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Second law analysis of counter flow cryogenic heat exchangers in presence of ambient heat-in-leak and longitudinal conduction through wall

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Abstract

Performance of highly effective heat exchangers is governed by the various internal and external irreversibilities. In low temperature applications, the performance of these heat exchangers strongly depends on the irreversibilities such as ambient heat-in-leaks, longitudinal heat conduction through separating wall of heat exchanger and conduction through high temperature connecting tubes when they are integrated to the system. The special focus of present analysis is the study of effect of these irreversibilities on the performance of heat exchangers through second law analysis. It is observed that the effect of ambient heat-in-leak is different for the balanced and imbalanced counter flow high NTU heat exchangers. Study also makes it possible to compare the different irreversibilities for varying range of NTU and analyze the influence of external irreversibilities on the performance of heat exchangers when either hot fluid or cold fluid is minimum capacity fluid.

Keywords: Counter flow heat exchangers; Second law analysis

1. Introduction

Heat exchangers used in cryogenic temperature range are subjected to huge temperature gradient inside the heat exchanger and from the ambient. These cryogenic heat exchangers can be differentiated from the conventional heat exchangers in two aspects, namely, the design point of view and the operational point of view. From the design point of view, several factors such as longitudinal conduction, ambient heat-in-leaks and variation of properties of fluid must be considered carefully, which are often unimportant in the design of conventional heat exchangers. The second aspect of these heat exchangers is related to the system operational requirement. In some conventional systems, the system will operate even the heat exchanger is having less than 50% effectiveness. In contrast, a cryogenic liquefier will produce no liquid if the heat exchanger effectiveness is less than approximately 85% [1]. However, these two aspects of cryogenic heat exchangers are complementary to each other.

Because of the practical importance, a large amount of literature is available to deal these secondary effects of heat leak from surrounding, longitudinal conduction through wall; heat loss through the cold end [2–7] from viewpoint of the first law of thermodynamics. In recent past, Gupta and Atrey [8] have published the numerical modeling considering the heat-in-leak from surrounding and longitudinal conduction through wall along with the experimental results for cryogenic applications.

The performance of these cryogenic heat exchangers, designed for high effectiveness, also depends on the integration of heat exchangers to the system. In many cryogenic

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Nomenclature

A	surface area of heat transfer (m^2)
A_c	cross-section area (m^2)
C	heat capacity rate of fluids defined by the product of $\dot{m}$ and C_p (W/K)
f	friction factor
G	mass velocity (kg/s m^2)
h	heat transfer coefficient (W/m^2 K)
$\dot{I}_n$	irreversibility number (W)
k	thermal conductivity of the wall material (W/m K)
L	length of heat exchanger (m)
$\dot{m}$	mass flow rate (kg/s)
M	non-dimensional parameter $= \frac{1}{v(\tau_1 - \tau_2)}$
n	number of transfer units for individual fluid stream
NTU	overall number of transfer units
N_f	non-dimensional friction number as defined in Eq. (34)
P	pressure (N/m^2)
ΔP	fluid pressure drop (N/m^2)
$\dot{Q}_a$	heat transfer from surrounding $= U_o A_o (T_a - T_c)$
R	dimensionless parameter defined as in Eq. (4)
$R_{1(2)}$	ideal gas constant (J/kg K)
$\dot{S}_{gen}$	entropy generation rate (W/K)
St	Stanton number
T	temperature (K)
T_0	ambient temperature (K)
U	overall heat transfer coefficient (W/m^2 K)
x	axial co-ordinate (m)
X	dimensionless axial co-ordinate in heat exchanger defined by x/L

Greek symbols

α	heat-in-leak parameter defined in Eq. (5)
θ	dimensionless temperature, defined as in Eq. (4)
ε	effectiveness of heat exchanger
λ, λ_1	dimensionless longitudinal conduction parameter, defined as in Eq. (5)
μ	heat capacity rate ratio (C_h/C_c)
v	heat capacity rate ratio (C_{min}/C_h)
ρ	density of fluid(s) (kg/m^3)
$\tau_{1,2}$	dimensionless fluid entrance temperature $= \frac{T_{h,in}\ (\text{or}\ T_{c,in}) - T_0}{T_0}$

Subscripts

a	ambient
c	cold fluid
h	hot fluid
i	inner
o	outer
out	outlet
w	wall
in	inlet
cv	control volume (heat exchanger)
cond.	contribution of end conduction
exit	contribution of cold fluid exit, leaving loss
ΔP	pressure drop contribution
surr.	contribution of ambient heat-in-leak
1	inner tube fluid
2	outer tube fluid

systems such as helium liquifier/refrigerator, the multiple heat exchangers have been used at different temperature levels. These heat exchangers are connected to the system through suitable connecting tubes. The heat conduction through these tubes from high temperature source could significantly affect the performance of heat exchanger in addition to ambient heat-in-leaks. These all such irreversibilities are known as *external irreversibilities* as the performance of heat exchanger is affected by the external means.

So, fact is that the performance of cryogenic heat exchanger is equally dependent on external irreversibilities in addition to internal irreversibilities and actual performance could be misleading if proper attention is not paid during the installation and operation of these heat exchangers. However, these external irreversibilities could be minimized by connecting the heat exchanger by using low conductivity materials with proper geometry and providing the proper insulation to the system in order to minimize ambient heat-in-leaks. Further, these minimized external irreversibilities could be accommodated in some cases by providing the additional NTU to heat exchanger in order to keep the performance with in range. However this additional NTU will increase the frictional pressure drop, so the exact trade off has to be worked out while designing of these heat exchangers. These different aspects of heat exchanger performance can be represented by single quantity, namely, the thermodynamic irreversibility.

From this point of view, thermodynamic analysis based on second law of thermodynamics is the most powerful tool for high effectiveness heat exchangers. This approach clearly indicates where improvements are possible and worthwhile. As high efficiency is the major requirement of cryogenic heat exchangers for lifting the energy from low temperature. Hence, it is worthwhile to minimize these sources of external irreversibilities in addition to internal irreversibilities to increase the efficiency of heat exchangers. In recent years, this thermodynamic optimization method has evolved as the *constructal law* of thermodynamics [9] which tells about the generation of the optimal architecture in flow systems.

The concept of irreversibility in heat exchanger analysis was introduced in literature by Bejan [10]. He explained the design methodology for a heat exchanger by using the second law of thermodynamics. He analyzed the two major components to exergy loss; viz., heat transfer across a finite temperature difference and fluid friction in greater detail and proposed a systematic methodology for computing the entropy generation through these two sources [11].

Since then, investigations in this direction were carried out by many others. Due to space limitation in this paper only few of them are mentioned here. Sarangi and Chowdhury [12] have investigated the entropy generation in counter flow heat exchanger and discussed about the mathematical consequences of this concept. Seculic [13] explained the meaning of the basic results of this concept for an arbitrary flow arrangement in a heat exchanger and examined the effect of NTU on the entropy generation level. For plate heat exchangers, an exergetic analysis was performed by Das and Roetzel [14,15]. They investigated the effect of axial dispersion through second law analysis. Recently, Hesselgreaves [16] has presented an analysis for rationalizing the second law analysis of heat exchangers.

However, most of these workers address the irreversibility caused by internal thermal resistance (ΔT) and frictional pressure drops (ΔP) and observed 'maximum entropy paradox'. These internal irreversibilities mentioned above are mainly important from the conventional heat exchanger design point of view. To the best knowledge of authors the effects of the external irreversibilities, from the cryogenic point of view, on heat exchanger performance through second law analysis have not been addressed in the literature. Alhough, Das and Roetzel [14,15] have considered the exergy loss due to the cold stream outlet, which is termed as a 'leaving loss' in this paper. However, they have disregarded the effect of ambient heat-in-leaks and have also not study the impact of imbalance on this exergetic loss.

In the present paper, the internal irreversibilities are reconsidered in presence of these external irreversibilities and expressions for external thermal irreversibilities due to heat-in-leak and end conduction are derived. Since axial conduction through separating wall, which is very important in high effective heat exchangers used in cryogenic applications, is added the more internal thermal irreversibility by decreasing the effectiveness of heat exchanger. Hence, the present paper also studies the effect of longitudinal conduction through separating wall of heat exchanger for the balanced and imbalanced flow of heat exchangers through second law analysis.

For this purpose the model presented earlier by Gupta and Atrey [8] for temperature response is used for calculating the exergy loss in terms of the irreversibility number. The exergetic losses due to these external irreversibilities with the degree of imbalance and their contributions to the total thermal irreversibilities have been studied in detail. The effect of heat-in-leak and axial conduction is examined in respect of second law and optimum range of NTU has been observed in presence of fluid friction for which heat exchanger can behave efficiently thermodynamically. The study is also extended to the imbalance flow heat exchanger, which is more common in cryogenic applications such as helium liquefier/refrigerators.

2. Temperature response of heat exchanger

The counter flow heat exchanger as shown in Fig. 1 can be modeled mathematically for the steady state behaviour in presence of longitudinal conduction and ambient heat-in-leaks [8]. Generally, in cryogenic applications such as helium liquefier/refrigerator, the low pressure cold stream flows in the shell side of heat exchanger. Therefore, the present analysis considered the cold stream of heat exchanger directly gets affected from ambient heat-in-leak as shown in Fig. 1.

The governing equations in non-dimensional form for temperature response of heat exchanger with heat-in-leak from surrounding and axial conduction through wall for the hot fluid, the cold fluid and wall are as follows:

$$\text{Hot fluid}: \frac{d\theta_h}{dX} + n_h(\theta_h - \theta_w) = 0 \tag{1}$$

$$\text{Wall}: \lambda v \frac{d^2\theta_w}{dX^2} + n_h(\theta_h - \theta_w) - \frac{n_c}{\mu}(\theta_w - \theta_c) = 0 \tag{2}$$

$$\text{Cold fluid}: \frac{d\theta_c}{dX} + n_c(\theta_w - \theta_c) - \alpha\mu v(\text{NTU})\theta_c = -\alpha\mu v(\text{NTU})(R+1) \tag{3}$$

In the above expressions, θ_h, θ_w and θ_c represent the dimensionless temperature of hot fluid, wall and cold fluid, respectively as given below. Following dimensionless parameters have been used in the above equations for analysis.

$$\theta = \frac{t - t_{c,in}}{t_{h,in} - t_{c,in}}, \quad R = \frac{t_a - t_{h,in}}{t_{h,in} - t_{c,in}}, \quad X = \frac{x}{L} \tag{4}$$

$$v = \frac{C_{min}}{C_h}, \quad \mu = \frac{C_h}{C_c}, \quad \alpha = \frac{U_o A_o}{U_i A_i}, \quad \lambda = \frac{kA_c}{C_{min} L} \tag{5}$$

$$n_h = \left(\frac{hA}{C}\right)_h, \quad n_c = \left(\frac{hA}{C}\right)_c, \quad \text{NTU} = \frac{U_i A_i}{C_{min}} \tag{6}$$

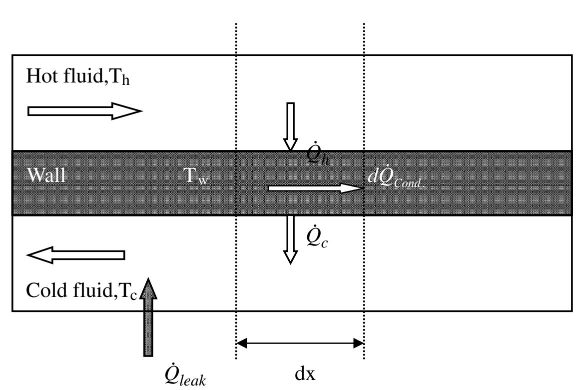

Fig. 1. Schematic of a heat exchanger mathematical model with heat-in-leak and longitudinal conduction through separating wall.

where n_h and n_c are the local heat transfer units of hot fluid and cold fluid streams, respectively; NTU is the total heat transfer units. The heat-in-leak parameter (α) is defined as the ratio of external conductance (U_oA_o) to the internal conductance (U_iA_i).

To determine the temperature profile across heat exchanger in presence of these secondary effects i.e. longitudinal conduction through wall and ambient heat-in-leaks, these energy equations of heat exchanger have divided in to n elements (typically 300) by using the finite difference method. These above equations are converted into linear algebraic equations and then solved by Gauss–Joardon method. Assuming adiabatic conditions for the wall at the two ends, following boundary conditions are applied:

$$X = 0: \qquad \theta_h = 1, \quad \frac{\partial \theta_w}{\partial X} = 0 \tag{7}$$

$$X = 1: \qquad \theta_c = 0, \quad \frac{\partial \theta_w}{\partial X} = 0 \tag{8}$$

It is assumed that the local NTU of each fluid stream is same; $n_h = n_c$ [8].

However, these governing equations are in non-dimensional form and can be interpreted for any temperature range but accuracy of the solution depends on thermo physical property variations with the temperature of the fluid in that range. In our case, the temperature of high pressure helium gas entering to the heat exchanger is 300.0 K and on the other hand the cold end entry temperature is 80.0 K. All the calculations performed in the present study are for the value of heat-in-leak parameter, $\alpha = 0.003$. The value of α is obtained empirically and supported by series of experimental investigations conducted by the present author [8]. However, the effect of the different values of α on the performance of the heat exchanger can be seen quantatively.

One of the temperature profiles calculated from this model is illustrated as an example in Fig. 2. The temperature profile obtained from the model will be used for calculating the entropy production in the heat exchanger by different sources.

2.1. Effectiveness of heat exchanger

The effectiveness of any heat exchanger is defined as the ratio of actual heat transfer to the maximum possible heat transfer. In normal conditions, the cooling effectiveness of hot stream, ε_h and heating effectiveness of cold stream, ε_c would be same. Although, when only the longitudinal conduction through separating wall is prominent in any heat exchanger with other negligible affects, the value of ε_h and ε_c would also be same but heat exchanger becomes less effective as shown in Fig. 2. However, in presence of heat-in-leaks from ambient in addition to longitudinal conduction, the cold fluid does not take up total heat transferred by the hot fluid stream. As a result of this, the effectiveness of heat exchanger based on hot fluid and cold fluid would not be equal.

Effectiveness on the basis of hot fluid and cold fluid can be expressed as follows:

$$\varepsilon_h = \frac{\dot{Q}_{hot}}{\dot{Q}_{max}} = \frac{C_h(t_{h,out} - t_{h,in})}{C_{min}(t_{h,in} - t_{c,in})} \tag{9}$$

$$\varepsilon_c = \frac{\dot{Q}_{cold}}{\dot{Q}_{max}} = \frac{C_c(t_{c,out} - t_{c,in})}{C_{min}(t_{h,in} - t_{c,in})} \tag{10}$$

The above expressions can be expressed in terms of dimensionless parameters as follows:

$$\varepsilon_h = (1 - \theta_{h,out})/\nu \tag{11}$$

$$\varepsilon_c = (\theta_{c,out})/\nu\mu \tag{12}$$

The effect of heat-in-leaks from surrounding can be calculated quantatively by applying the energy balance over the whole heat exchanger and can be described as follows:

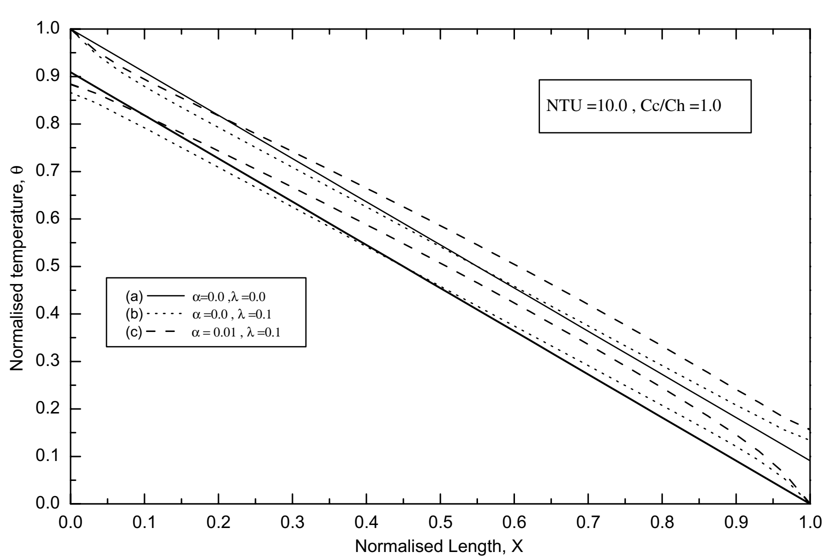

Fig. 2. Temperature profile of the hot and cold streams in presence of the longitudinal conduction through separating wall and heat-in-leak from ambient.

$$\dot{Q}_a = \dot{Q}_c - \dot{Q}_h \tag{13}$$

Eq. (13) can be presented in dimensionless form by dividing the $\dot{Q}_{max}$ as follows:

$$\phi = \varepsilon_c - \varepsilon_h \tag{14}$$

3. Second law analysis

It is said that the traditional methods for analysis and design of heat exchanger using first law of thermodynamics emphasized that the energy is conserved quantity wise and disregards the quality of energy. It means it takes no account of wastage of useful energy (available energy) during the heat transfer process. Conventional approach recognizes only the total amount of energy supplied to the system and as a result, this yields the substantive design rather than the thermodynamically efficient one. In the second law analysis all loses are treated as the source of entropy production. It is thus possible to compare and sum them. Therefore, second law of thermodynamics makes possible to design a heat exchanger, which operates in most efficient way thermodynamically and wasting the least amount of energy.

The irreversibilities in any heat exchanger may be listed as follows:

(1) Internal irreversibility;
(2) External irreversibility.

The internal irreversibility in heat exchanger is caused by the finite temperature difference between two heat exchange streams and by the internal dissipative effects like friction. Heat transfer between two streams through an infinitesimal temperature difference is regarded as reversible. So to transfer a finite amount of heat through infinitesimal temperature difference would require an infinite thermal size (infinite NTU). Hence all actual heat transfer processes are through a finite temperature difference and are, therefore, irreversible in nature. This irreversibility in heat exchanger may also be termed as *internal thermal irreversibility*.

In all traditional approaches, heat exchanger is considered perfectly insulated from ambient. But in actual practice it is not worthwhile, especially in high efficient cryogenic heat exchangers, to disregard the heat-in-leak from surrounding that causes some amount of destruction of useful energy. The other sources of the irreversibility in cryogenic heat exchangers are the cold end and hot end of heat exchanger. The cold end of heat exchanger where hot stream, which is cooled within the heat exchanger, leaves for the process application and exergy associated with it is a 'useful energy' and not wastage. Hence, the cold end may provide the room for moving the irreversibility via conduction through connecting tubes where it has to be used. At the hot end of heat exchanger, the cold stream, after cooling the hot stream, does not approach the hot stream inlet temperature due to the internal irreversibility. Hence the exergy left with the cold gas, which the exchanger has failed to transfer to hot fluid, having gone waste to the surroundings since any thing external to it forms part of surroundings. The exergy loss associated to the hot end may be termed as the 'leaving exergy losses' of heat exchangers. The irreversibilities listed in this paragraph are called the external irreversibility. However, it is more convincing to describe these irreversibilities as the *external thermal irreversibilities*. In brief, it indicates that the heat exchanger with same thermal size (NTU) and for given operating parameters may behave differently due to these external thermal irreversibilities. Therefore, it makes good engineering sense to focus on external irreversibilities in addition to internal thermal irreversibilities for the effective operation of heat exchanger. Our concern is to describe the fundamental approach as well as practical importance of problem to optimize the heat exchanger performance by minimizing these external and internal thermal irreversibilities and hence minimization of entropy generation is the hidden concept of good engineering design.

The irreversibility associated with any process, which is the quantitative measure of exergy loss in the process, is related to the entropy production within the system. This can be presented by Gouy–Stodola theorem as follows:

$$\dot{I} = T_0 \dot{S}_{gen} \tag{15}$$

As has been said that entropy generated with in the heat exchanger can be split as follows:

$$\dot{S}_{gen} = \dot{S}_{gen,internal} + \dot{S}_{gen,external} + \dot{S}_{gen,\Delta P} \tag{16}$$

3.1. Thermal irreversibility

The internal thermal irreversibility as shown in Fig. 3 is associated with the entropy generation due to stream-to-stream temperature difference within the control volume and can be represented as

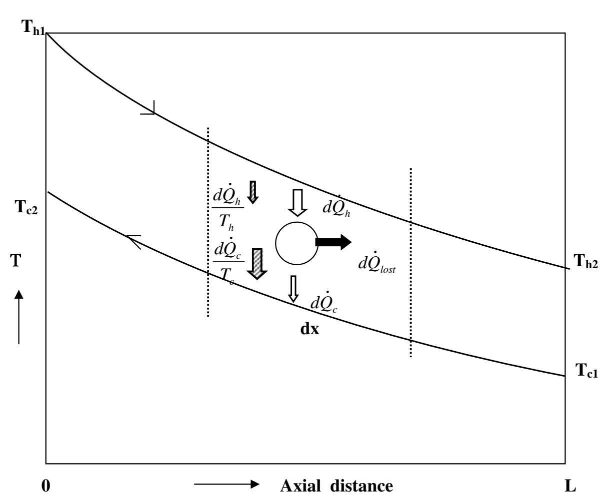

Fig. 3. Internal irreversibility in heat exchanger.

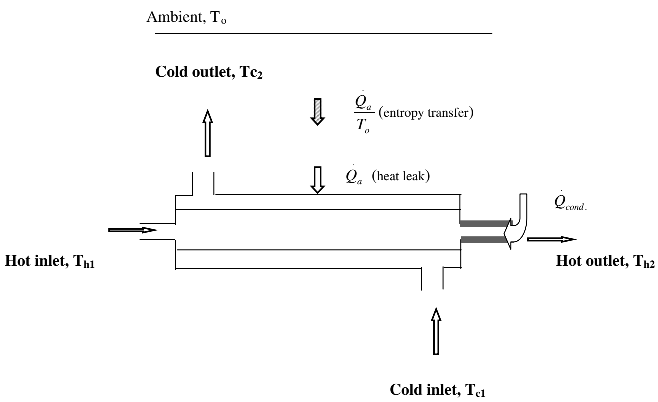

Fig. 4. Heat exchanger subjected to the external irreversibilities.

$$\dot{S}_{\text{gen,internal}} = \dot{S}_{\text{gen,cv}} \tag{17}$$

The external thermal irreversibilities as demonstrated in Fig. 4 is associated with the entropy generation due to heat-in-leak from surrounding, exergy left with the cold fluid outlet termed as 'leaving exergy loss' and due to heat conduction through high temperature source at the cold end where the cooled hot fluid has to be used. This can be expressed as

$$\dot{S}_{\text{gen,external}} = \dot{S}_{\text{gen,surr.}} + \dot{S}_{\text{gen,exit}} + \dot{S}_{\text{gen,cond.}} \tag{18}$$

Assuming the cryogens as an ideal gases, the entropy balance over the heat exchanger and surrounding in terms of irreversibility equations as follows:

$$\dot{I}_{\text{internal}} = \left[(\dot{m}c_p)_\text{h} \ln\frac{T_{\text{h2}}}{T_{\text{h1}}} + (\dot{m}c_p)_\text{c} \ln\frac{T_{\text{c2}}}{T_{\text{c1}}}\right] T_0 \tag{19}$$

$$\dot{I}_{\text{external}} = \left[\frac{\dot{Q}_\text{a}}{T_0} + \left\{(\dot{m}c_p)_\text{c} \ln\frac{T_0}{T_{\text{c2}}} + (\dot{m}c_p)_\text{c}\frac{T_{\text{c2}} - T_0}{T_0}\right\} + \dot{Q}_{\text{cond.}}\frac{T_0 - T_\text{w}}{T_0 T_\text{w}}\right] T_0 \tag{20}$$

where T_w is the wall temperature at the cold end and is assumed to be the average temperature of the cold end.

Many investigators have defined the fractional exergy loss differently. But in this paper it is defined by dividing the maximum heat that can be transferred ideally so that one can get better understanding about the quantative values of exergy loss in respect of maximum heat transferred ($\dot{Q}_{\max}$). It can be termed as irreversible number ($\dot{I}_\text{n}$).

$$(\dot{I}_\text{n}) = \frac{\dot{I}}{\dot{Q}_{\max}} \tag{21}$$

where $(\dot{I}_\text{n}) = (\dot{I}_\text{n})_{\text{cv}} + (\dot{I}_\text{n})_{\text{surr.}} + (\dot{I}_\text{n})_{\text{exit}} + (\dot{I}_\text{n})_{\text{cond.}}$

$$(\dot{I}_\text{n})_{\text{cv}} = M \ln\left[1 - \frac{\varepsilon_\text{h}}{M(\tau_1 + 1)}\right] + \frac{M}{\mu}\ln\left[\frac{(\varepsilon_\text{h} + \phi)\mu}{M(\tau_2 + 1)} + 1\right] \tag{22}$$

$$(\dot{I}_\text{n})_{\text{surr.}} = \phi \tag{23}$$

$$(\dot{I}_\text{n})_{\text{exit}} = \left[-\frac{M}{\mu}\ln\left\{\frac{(\varepsilon_\text{h} + \phi)\mu}{M} + \tau_2 + 1\right\}\right] + (\varepsilon_\text{h} + \phi) + \frac{\tau_2}{\mu}M \tag{24}$$

$$(\dot{I}_\text{n})_{\text{cond.}} = \frac{\lambda_1 v[\varepsilon_\text{h} - (\tau_1 + \tau_2)M]^2}{2[2M(\tau_1 + 1) - \varepsilon_\text{h} - v^{-1}]} \tag{25}$$

3.2. Pressure drop irreversibility

Any heat exchanger design involves the consideration of both heat transfer rate between fluids and mechanical pumping power extended to overcome the fluid friction during the fluid flow through heat exchanger. In many heat exchanger designs, especially for low-density fluids such as gases, the frictional power expenditure is considerable. In addition to the compressor power requirement, like in conventional heat exchanger, the pressure drop design for cryogenic heat exchangers is extremely important from the point view of overall performance of the cryogenic systems. The tube side pressure drop across the heat exchanger will reduce the amplitude of high pressure stream thereby reducing the area of expansion space in PV diagram and the gross refrigeration produced by the refrigerator/liquefier.

On the other hand, pressure drop in the shell side is extremely important for any cryogenic systems such as helium liquefier/refrigerator. In a helium liquefier/refrigerator (helium normal boiling point 4.2 K and critical pressure 2.2 bar), the total pressure drop of the shell side should not be more than 0.2 bar because of the constraint of critical pressure of helium.

Hence, the magnitude of this pressure drop irreversibility is extremely important in cryogenic heat exchangers. With the ideal gas assumption the entropy generation due to pressure drop can be computed as follows:

$$(\dot{S}_{\text{gen}})_{\Delta P} = \dot{m}_\text{h} R_1 \ln\frac{P_{1,\text{in}}}{P_{1,\text{out}}} + \dot{m}_\text{c} R_2 \ln\frac{P_{2,\text{in}}}{P_{2,\text{out}}} \tag{26}$$

$$\frac{P_{\text{in}}}{P_{\text{out}}} = 1 + \frac{\Delta P}{P_{\text{out}}} \tag{27}$$

or

$$\ln\left(1 + \frac{\Delta P}{P_{\text{out}}}\right) \approx \frac{\Delta P}{P_{\text{out}}} \tag{28}$$

The pressure drop in compact heat exchangers can be given as [17]

$$\frac{\Delta P}{P} = f \cdot \frac{4L}{D} \cdot \frac{G^2}{2\rho P} \tag{29}$$

since

$$\text{NTU} = \frac{4L}{D} St \tag{30}$$

Eq. (29) can be written as

$$\frac{\Delta P}{P} = f \cdot \frac{G^2}{2\rho P} \cdot \frac{\text{NTU}}{St} \tag{31}$$

Eq. (26) can be represented in the following form:

$$\dot{S}_{\mathrm{gen},\Delta P} = \dot{m}_{\mathrm{h}} R_1 \cdot \frac{f_1 G_1^2}{2\rho_{\mathrm{h}} P_{\mathrm{h,out}}} \cdot \frac{n_{\mathrm{h}}}{St_1} \cdot \frac{c_{\mathrm{h}}}{c_{\mathrm{h}}} + \dot{m}_{\mathrm{c}} R_2 \cdot \frac{f_2 G_2^2}{2\rho_{\mathrm{c}} P_{\mathrm{c,out}}} \cdot \frac{n_{\mathrm{c}}}{St_2} \cdot \frac{c_{\mathrm{c}}}{c_{\mathrm{c}}} \tag{32}$$

The above Eq. (32) can be simplified and can be written as

$$\dot{S}_{\mathrm{gen},\Delta P} = \dot{m}_{\mathrm{h}} \cdot c_{\mathrm{h}} n_{\mathrm{h}} N_{\mathrm{f1}}^2 + \dot{m}_{\mathrm{c}} c_{\mathrm{c}} n_{\mathrm{c}} N_{\mathrm{f2}}^2 \tag{33}$$

where

$$N_{\mathrm{f}}^2 = \frac{fG^2}{2\rho PSt} \cdot \frac{R}{C} \tag{34}$$

The above Eq. (33) can be represent in non-dimensional irreversibility number as follows:

$$\dot{I}_{\mathrm{n},\Delta P} = \frac{1}{v(\tau_1 - \tau_2)} [n_{\mathrm{h}} N_{\mathrm{f1}}^2 + n_{\mathrm{c}} N_{\mathrm{f2}}^2 \mu^{-1}] \tag{35}$$

4. Results and discussion

Performance of cryogenic heat exchangers designed for high effectiveness is strongly influenced by the operational parameters as well as the integration of heat exchanger with the systems. One such operational parameter which can degrade the performance of heat exchanger is ambient heat-in-leak. Gupta and Atrey [8] have shown experimentally that the cold end of heat exchanger is severely affected by the ambient heat-in-leaks. The study clearly indicates that a heat exchanger designed for high effectiveness may fail to deliver the desired performance. However, these experimental studies were limited for the balanced flow heat exchangers.

The following analysis in preceding sections are presented to analyze these external effects described earlier in addition to the internal irreversibilities on the exergetic performance of the balanced and unbalanced counter flow heat exchangers in cryogenic temperature range. All calculations are performed for the heat-in-leak parameter $\alpha = 0.003$, however, this may taken any number to analyze the effect of this parameter and it is in no way the mathematical constraints. Generally, it varies from 0.001 to 0.003 for cryogenic temperature range [8]. The cryogenic heat exchangers generally experience the imbalance flow with in the heat exchangers. This imbalance may be either due to the hot fluid is minimum capacity fluid or the cold fluid is minimum capacity fluid. In some cryogenic systems like a helium liquefier, where series of heat exchangers are used, both types of unbalancing are exist. In the present study, the values of μ are taken 0.95, 1.0, 1.1. These values are chosen to address the above mentioned situations of helium liquefier where some critical aspects of the performance of heat exchangers are immensely influenced by these parameters.

Fig. 5 demonstrates the effect of imbalance on the external thermal irreversibilities, $\dot{I}_{\mathrm{n,external}}$ for the value of NTU = 5.0. As stated in previous section, the external thermal irreversibilities, $\dot{I}_{\mathrm{n,external}}$ has three components; ambient heat-in-leak, cold stream exit loss (or leaving loss), and end conduction from high temperature source. In general, it can be concluded from the figure that the contribution of external irreversibilities is higher when hot fluid is minimum capacity fluid. The cold stream exit exergy loss (leaving loss) is much higher when hot fluid is minimum capacity fluid ($C_{\mathrm{h}}/C_{\mathrm{c}} < 1.0$). It shows an exponential decrease as the heat capacity ratio increases and becomes non-significant when the heat capacity ratio ($C_{\mathrm{h}}/C_{\mathrm{c}}$) becomes greater than 1.0. In the case, when heat capacity ratio is greater than 1.0, the cold stream exit temperature will have less temperature difference, depends on degree of imbalance, from the ambient temperature T_0 (300.0 K) and cold stream will have less energy to dump in to the ambient. Hence, the exergetic loss of heat exchanger is very minimal in this case. Moreover, it can be concluded from

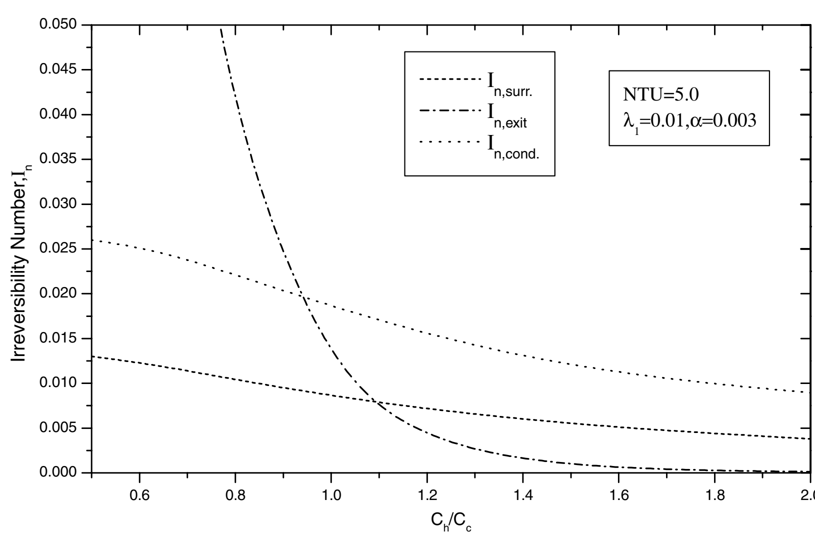

Fig. 5. Effect of $C_{\mathrm{h}}/C_{\mathrm{c}}$ on various external irreversibilities.

Fig. 5 that this component of external irreversibility is much significant when the hot fluid of fixed NTU heat exchanger is minimum capacity fluid ($C_h/C_c < 1.0$).

Similarly, the irreversibility number for the end conduction loss and ambient heat-in-leak shows a monotonous decrease with the heat capacity ratio. The ambient heat-in-leak is directly exposed to the cold fluid of heat exchanger and the amount of heat-in-leaks is governed by the temperature difference between the surrounding and heat exchanger for the particular surrounding thermal conduction. As the heat capacity ratio (C_h/C_c) is shifted towards the higher side of 1.0, then the average temperature of cold fluid shifted towards the higher side and reduces the temperature difference across heat exchanger and ambient. Hence, the exergetic performance will improve with an increase in heat capacity ratio. The exergetic performance of heat exchanger due to the end conduction is also improved, when the cold fluid is minimum capacity fluid. However, the exergetic loss is higher for the conduction from the end as compared to the ambient heat-in-leak for the given conditions. Therefore, the exergy loss due to this component is equally important for the performance point of view of cryogenic heat exchangers. Every care has to be taken during integration of heat exchanger to the cryogenic system to keep this loss minimum.

Figs. 6–8 present the effect of individual external irreversibility and their simultaneous effect as a function of NTU for $\alpha = 0.003$, $\lambda_1 = 0.01$ and $C_h/C_c = 0.95$, 1.0, 1.1. In all these figures, the irreversibility due to the exit cold fluid is vary exponentially with the NTU of heat exchanger.

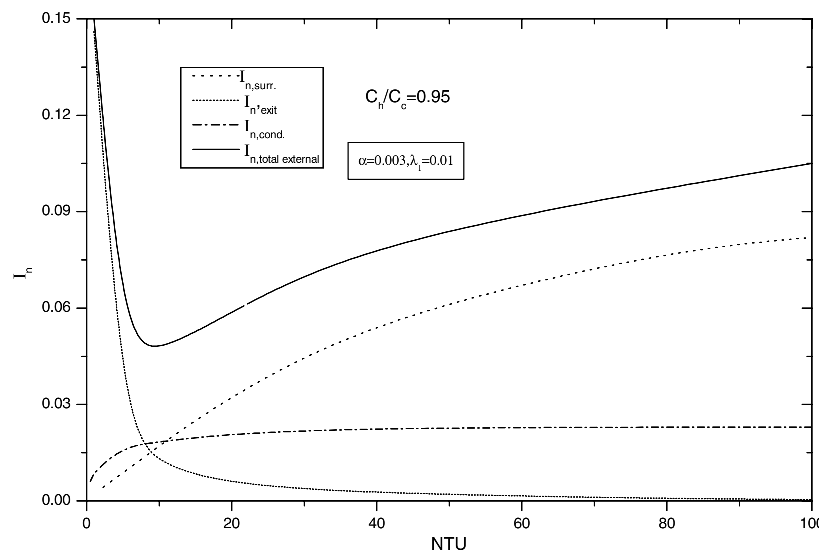

Fig. 6. Variation in individual and combined external irreversibilities with NTU for $C_h/C_c = 0.95$.

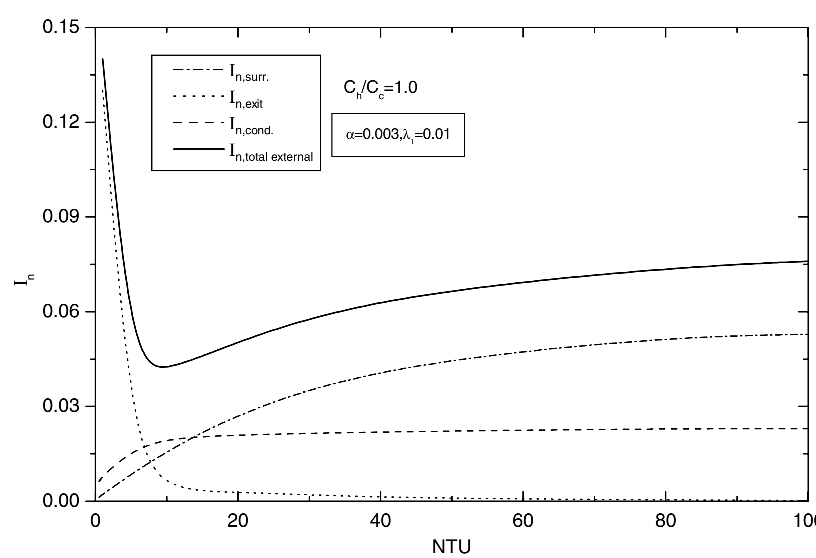

Fig. 7. Variation in individual and combined external irreversibilities with NTU for $C_h/C_c = 1.0$.

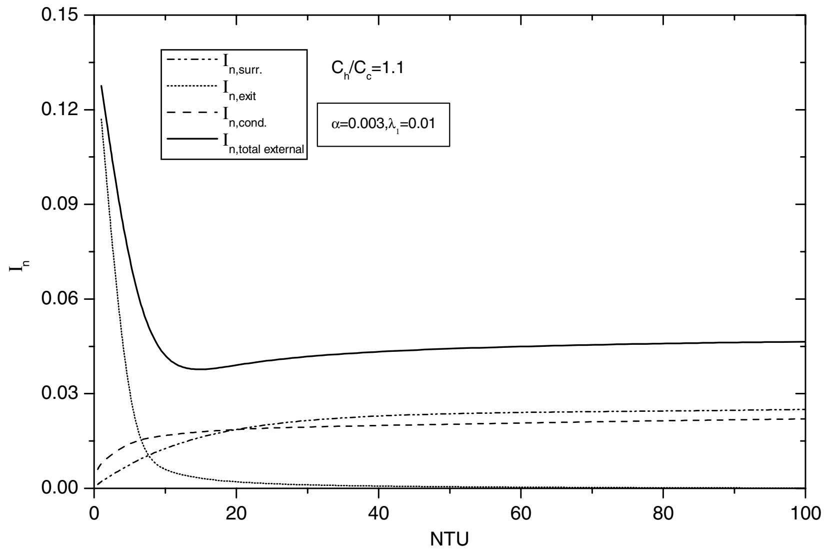

Fig. 8. Variation in individual and combined external irreversibilities with NTU for $C_h/C_c = 1.1$.

It becomes minimum as NTU of heat exchanger becomes more than 10.0 for the given conditions. The degree of this irreversibility depends on how much the temperature difference of exit cold fluid with the surrounding. In balanced flow heat exchanger ($C_h/C_c = 1.0$), as NTU increases, the temperature difference of the exit cold fluid with the surrounding reduces, hence the irreversibility will decrease with NTU. Similar trend will be observed when the $C_h/C_c = 1.1$, due to the close match of the cold fluid exit temperature with the surrounding. But when the value of C_h/C_c is 0.95, the hot fluid minimum capacity fluid, the exit temperature of the cold fluid will have more temperature difference with the surrounding as the cold fluid will experience less temperature change as compare to other cases described here for the same NTU. Thus, the irreversibility will be higher in this case.

The external irreversibility due to the end conduction from high temperature source (i.e. connectors) is increases up to the value of NTU = 10.0 then it becomes almost constant as NTU increases in all the cases as shown in Figs. 6–8. This is due to the facts that the separating wall temperature becomes nearly constant as the NTU of the heat exchanger increases for the same operating conditions. Therefore, the temperature gradient established between the hot end of heat exchanger and high temperature source becomes constant. It can also be observed from these figures that the effect of ambient heat-in-leak is lowest when the cold fluid is minimum capacity fluid ($C_h/C_c = 1.1$) than other two cases as shown in Figs. 6 and 7 and the irreversibility due to this ambient heat-in-leak nearly becomes constant as the NTU of heat exchanger increases after the value of 20.0. On the other hand, the irreversibility due to heat-in-leak from ambient is increases with NTU for other two cases presented in Figs. 6 and 7. However, the irreversibility due to ambient heat-in-leak is higher when the value of $\mu = 0.95$ than for the value of $\mu = 1.0$.

The combined effect of these external irreversibilities can also be analyzed from these Figs. 6–8. Fig. 8 shows that the combined effect of these external irreversibilities first decrease sharply as the NTU increases up to 10.0 then it becomes constant as the NTU of heat exchanger increases for the value of $\mu = 1.1$. This may be due to the fact that the irreversibility due to the exit cold fluid is almost becomes negligible at the value of the NTU = 10.0 and the other irreversibilities shown in figure are almost becomes constant after this value of NTU. One important physical conclusion can be drawn from this figure that the increase in NTU after 10.0 would not be beneficial to compensate these irreversibilities for this case. While for the cases of $\mu = 1.0$, 0.95 as shown in Figs. 6 and 7, the total external thermal irreversibility is first decreases sharply up to the value of NTU is equal to 10.0 and then it goes on increasing with the NTU. This minima observed in these two cases may be due the fact that the irreversibility due to ambient heat-in-leak is increases as the NTU of heat exchanger increases. The rising effect of this irreversibility offset the effect of the diminishing irreversibility with the NTU due to other factors. Therefore, one has to be careful while designing the imbalanced counter flow heat exchangers for cryogenic applications due to the impact of the external irreversibilities on the performance of heat exchangers.

Fig. 9 presents the contribution of external and internal thermal irreversibilities to the total thermal irreversibilities for the optimum NTU of heat exchanger. Figure shows that for the heat capacity ratio (C_h/C_c) of 0.6, the contribution of the internal thermal irreversibility to the total thermal irreversibility is 75% and remaining 25% is contributed by the external thermal irreversibilities. On the other hand, when the heat capacity ratio (C_h/C_c) is 2.0, the contribution of internal thermal irreversibility to the total thermal irreversibility is 98% and remaining 2% is only contributed

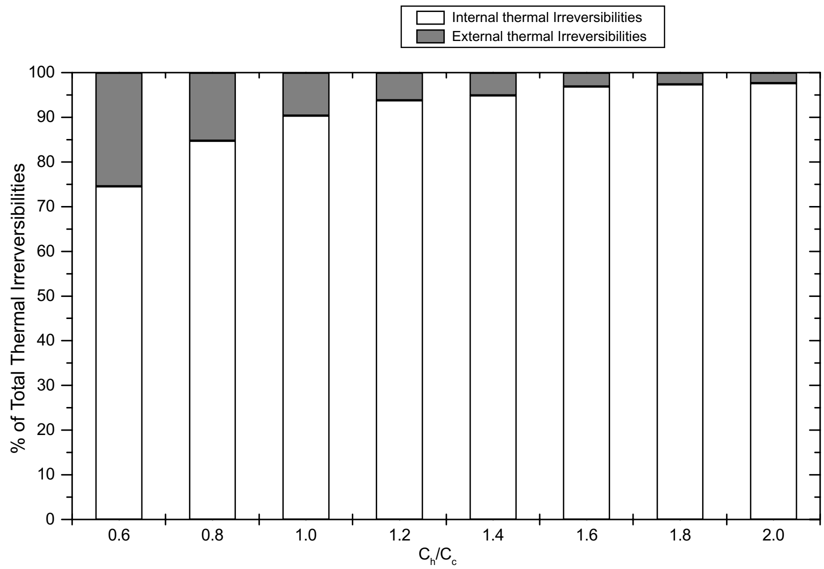

Fig. 9. Contribution of the internal and external irreversibilities to the total thermal irreversibilities for the optimum value of NTU.

by the external thermal irreversibilities. Therefore, it can be concluded from figure that the contribution of the external irreversibilities is different for the cases when hot fluid is minimum capacity fluid or when the cold fluid is minimum capacity fluid. In some practical applications like helium liquefiers/refrigerators, where a series of heat exchangers are used, some heat exchangers are experienced the cold fluid is minimum capacity fluid ($C_h/C_c > 1$) whether some heat exchangers are experienced the hot fluid is minimum capacity fluid ($C_h/C_c < 1$).The effect of these external irreversibilities has to be considered accordingly during the design stage only. One important conclusion can be drawn from the figure that as the degree of imbalance shifted below unity of heat capacity ratio (C_h/C_c), the effect of the external thermal irreversibilities becomes significant and they become less important from the design point of view when the degree of imbalance shifted towards higher than the unity.

Fig. 10 shows the effect of ambient heat-in-leak and longitudinal conduction through separating wall on the internal thermal irreversibility, resulting from the finite temperature difference, of the heat exchanger as a function of the NTU. Figure shows that the internal thermal irreversibility ($\dot{I}_{n,\Delta T}$) undergoes through a maxima at the NTU which gives the effectiveness of 50%. The maxima of entropy generation at the effectiveness of 50% are also observed by other investigators [10,12,13] in their analysis for conventional heat exchangers. Bejan calls this behav-

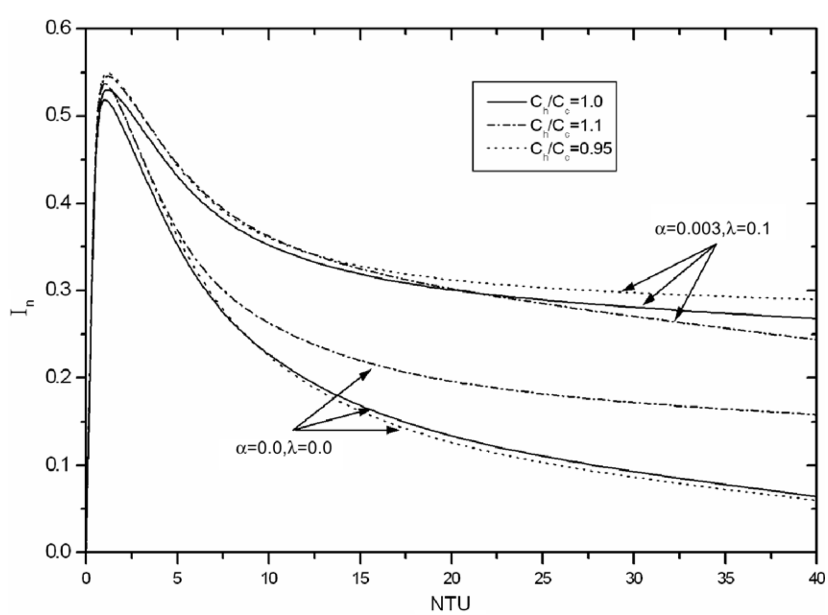

Fig. 10. Effect of ambient heat-in-leak and longitudinal conduction on internal thermal irreversibilities.

iour as the 'entropy generation paradox'. However, this maxima zone is not valid for the counter flow cryogenic heat exchangers for effectiveness point of view as the most of the cryogenic systems required the effectiveness more than 95%.

So, fact is that the entropy generation behaviour is more important in the high NTU range for the cryogenic applications. Fig. 10 shows that the internal thermal irreversibility ($\dot{I}_{n,\Delta T}$) goes on decreasing (after passing the maxima) with the NTU for the case of balance flow (μ =1.0) in the absence of the longitudinal conduction and ambient heat-in-leak while it becomes almost constant in the case of the imbalance flow ($\mu = 0.95$, 1.1) of the heat exchanger. However, in this case ($\alpha = 0.0$, $\lambda = 0.0$) the heat exchanger will be more irreversible, when cold fluid will be minimum capacity fluid. These curves of the figure reconfirms the fact that the imbalance heat exchanger will always subject to some irreversibilities whatever the large size of the heat exchanger as also noted by Bejan [18] in his analysis.

On the other hand, when the heat exchanger experienced the longitudinal conduction through separating wall and ambient heat-in-leak, the irreversibility is more as compare to the absence of these effects ($\alpha = 0.0$, $\lambda = 0.0$) in each of the case study in this figure. The irreversibility in heat exchanger is more when the hot fluid is minimum capacity fluid after the value of NTU = 20, which is the contradictory observation in respect of the heat exchanger where these secondary losses are negligible ($\alpha = 0.0$, $\lambda = 0.0$). In the cases of $\mu = 0.95$, 1.0, the irreversibility becomes almost constant with NTU after the value of 20.0 while for $\mu = 1.1$ (cold fluid, minimum capacity fluid) it shows a decreasing trends with the NTU. As the cold fluid is minimum capacity fluid, the average temperature of the cold fluid will be higher and the ambient heat flow will be less for this case. Hence, increase in NTU is offsetting the effect of ambient heat-in-leak and longitudinal conduction and showing the decreasing trend for this case. Here, it can be concluded that heat exchanger behaves differently when the hot fluid is minimum capacity fluid than the cold fluid is minimum capacity fluid in presence of these secondary effects. It is therefore important to analyze the effect of these secondary losses in both cases of minimum capacity fluid, hot and cold fluid.

Fig. 11 presents the effect of the longitudinal conduction through separating wall of heat exchanger, end conduction through high temperature source and ambient heat-in-leak on the total thermal irreversibilities as a function of the NTU. The figure also highlights the effect of imbalance on the total thermal irreversibilities. It can be observed from figure that the maximum's disappears if one considers the external thermal irreversibilities along with the internal thermal irreversibilities as also observed by Das and Roetzel [14,15]. Fig. 11 shows the optimum range (10.0–20.0) of the NTU in presence of the irreversibilities caused by longitudinal conduction and ambient heat-in-leak for designing the cryogenic balance and imbalance flow heat exchangers. However, figure shows the optimum value of NTU in the absence of pressure drop irreversibility. Although, it can be restated here that the total thermal irreversibility, when only the irreversibility caused by the exit cold fluid and the end conduction from the high temperature source is added to the internal irreversibility, is either decreases or becomes constant with the NTU depends on the nature of the imbalance.

The irreversibility due to pressure drop is equally important for cryogenic heat exchangers as mentioned earlier. However, the irreversibility due to pressure drop may not be important for the conventional heat exchangers from operational point of view. Therefore, it is omitted by the many investigators in their analysis [12,13]. Fig. 12 shows

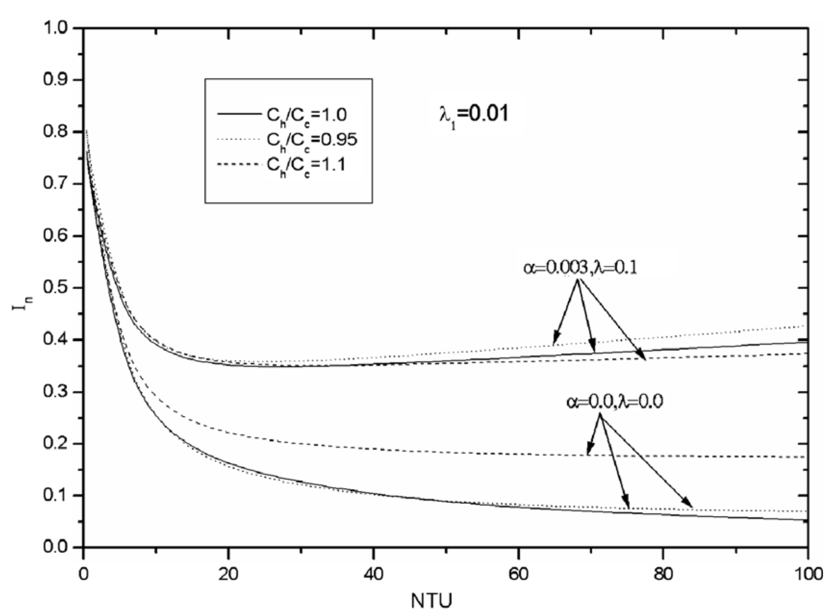

Fig. 11. Effect of ambient heat-in-leak and longitudinal conduction on total thermal irreversibilities.

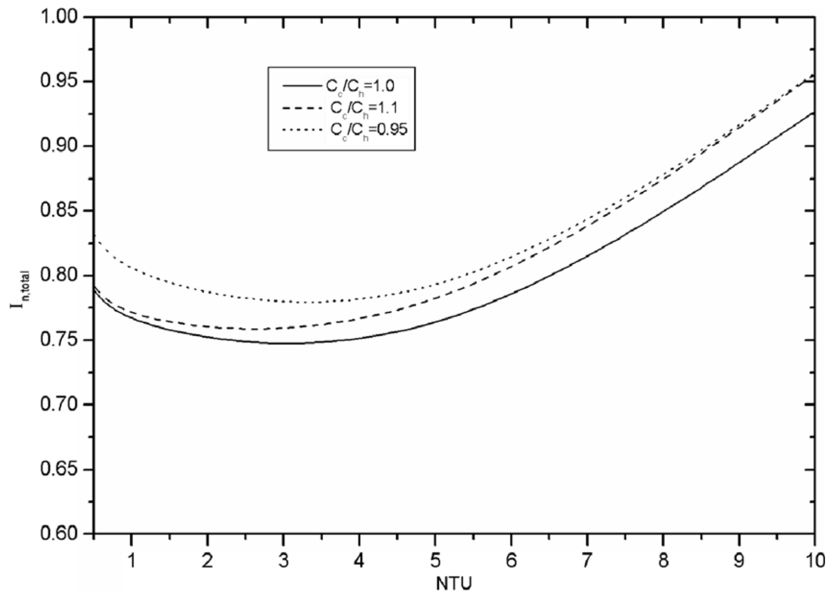

Fig. 12. Effect of imbalance in presence of ambient heat-in-leak and longitudinal conduction on total irreversibility of the heat exchanger.

the total irreversibility for different heat capacity ratios for a given friction parameter of $N_f = 0.1$. Whatever the value of the heat capacity ratio (C_h/C_c), total irreversibility always shows a definite minima. This is due to the competing nature of the thermal and fluid frictional irreversibilities with increase in the NTU as observed by Bejan [18]. However, the minimum irreversibility is different for the different value of C_h/C_c as shown in figure. This may be due to the fact that as the heat capacity ratio changed, the NTU requirement for the given heat duty is changed. Therefore, the pressure drop irreversibility is changed with the change in NTU for the given operating conditions. It is interesting to note that it is always possible to operate a heat exchanger in the minimum irreversible zone with the range of NTU and thus provide more flexibility to the designer. In all these examples it is assumed that the counter flow heat exchanger is operating between 300 and 80 K temperature ranges.

5. Conclusion

An attempt has been made to address the various external irreversibilities of the counter flow cryogenic heat exchangers in addition to the internal irreversibilities through second law analysis. In many cryogenic applications such as helium liquefier/refrigerator, both balanced and imbalanced flow heat exchangers are used and the performance of these heat exchangers severely influenced by these external irreversibilities in addition to other parameters. Keeping of this in mind, the effect of these irreversibilities has been studied for both balanced and imbalanced flow of heat exchangers. It can be observed from the present analysis that the effect of the external thermal irreversibilities is more when the hot fluid is minimum capacity ($C_h/C_c < 1.0$) fluid and the contribution of the total external irreversibilities to the total thermal irreversibilities is only 2% when the value of C_h/C_c is 2.0. It can therefore be concluded that in the absence of pressure drop irreversibility, the performance of heat exchanger is less susceptible to the external irreversibilities for the given heat exchanger configuration in the case when the cold fluid is minimum capacity fluid.

In the present analysis, the variation of internal thermal irreversibilities with the NTU in presence of ambient heat-in-leak and longitudinal conduction through separating wall of heat exchanger is also studied for the balanced and imbalanced flow of heat exchangers. It is observed that variation in exergetic loss with the NTU is different for the different values of μ in presence of ambient heat-in-leak and longitudinal conduction as compare to the value of $\alpha = 0.0$, $\lambda = 0.0$ for the same value of μ. The internal exergetic loss is decreases as the NTU of heat exchanger increases when cold fluid is minimum capacity fluid while the internal exergetic loss becomes nearly constant as the NTU of heat exchanger increases when hot fluid is minimum capacity fluid. The practical conclusion is that the increase in NTU is not always beneficial from the exergetic performance point of view in presence of external irreversibility for the design of the heat exchanger with negligible pressure drop. The analysis presented here also reconfirms that there is optimum range of NTU exist in presence of the external irreversibilities and internal irreversibilities where heat exchanger can perform efficiently thermodynamically.

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